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Cosmological Constant Versus Quintessence

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The mounting evidence that the universe is presently undergoing accelerating expansion has restored some credit to the scenarios with a nonvanishing cosmological constant. From the point of view of a theory of fundamental interactions, one may argue that a dynamical component with negative pressure is easier to achieve. As an illustration, the quintessence scenario is described and its shortcomings are discussed in connection with the nagging "cosmological constant problem."

1. COSMOLOGICAL CONSTANT

The cosmological constant appears as a constant in the Einstein equation

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -8\pi G_N T_{\mu\nu} + \lambda g_{\mu\nu} \tag{1}$$

where G_N is Newton's constant and $T_{\mu\nu}$ is the energy-momentum tensor. The cosmological constant λ is thus of the dimension of an inverse length squared. It was introduced by Einstein [1] in order to build a static universe model, its repulsive effect compensating the gravitational attraction, but, as we now see, constraints on the expansion of the universe impose for it a very small upper value.

It is more convenient to work in the specific context of a Friedmann universe with a Robertson metric

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right] \tag{2}$$

where $a(t)$ is the cosmic scale factor. Implementing energy conservation into the Einstein equation then leads to the Friedmann equation, which gives an expression for the Hubble parameter H:

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$$H^2 \equiv \frac{\dot{a}^2(t)}{a^2(t)} = \frac{1}{3}\,(\lambda + 8\pi G_N \rho) - \frac{k}{a^2} \tag{3}$$

where, using standard notations, $\dot{a}$ is the time derivative of the cosmic scale factor, $\rho = T^0{}_0$ is the energy density, and the term proportional to k is a spatial curvature term [see (2)]. Note that the cosmological constant appears as a constant contribution to the Hubble parameter.

Evaluating each term of the Friedmann equation at present time allows for an estimation of an upper limit on λ. Indeed, we have $H_0 = h_0 \times 100$ km s^{-1} Mpc^{-1} with h_0 of order one, whereas the present energy density ρ_0 is certainly within one order of magnitude of the critical energy density $\rho_c = 3H_0^2/(8\pi G_N) = h_0^2 \times 2 \cdot 10^{-26}$ kg m^{-3}; moreover, the spatial curvature term certainly does not represent presently a dominant contribution to the expansion of the universe. Thus, (3) implies the following constraint on λ:

$$|\lambda| \leq H_0^2 \tag{4}$$

In other words, the length scale $l_\Lambda \equiv |\lambda|^{-1/2}$ associated with the cosmological constant must be larger than $H_0^{-1} = h_0^{-1} \times 10^{26}$ m, and thus a macroscopic distance.

This is not a problem as long as one remains classical. Indeed, H_0^{-1} provides a natural macroscopic scale for our present universe. The problem arises when one tries to combine gravity with quantum theory. Indeed, from Newton's constant *and* the Planck constant $\hbar$ one can construct a mass scale or a length scale

$$m_P = \sqrt{\frac{\hbar c}{8\pi G_N}} = 2.4 \times 10^{18}\ \mathrm{GeV}/c^2$$

$$l_P = \frac{\hbar}{m_P c} = 8.1 \times 10^{-35}\ \mathrm{m}$$

The above constraint now reads

$$l_\Lambda \equiv |\lambda|^{-1/2} \geq 1/H_0 \sim 10^{60}\, l_P \tag{5}$$

In other words, there are more than 60 orders of magnitude between the scale associated with the cosmological constant and the scale of quantum gravity.

A rather obvious solution is to take $\lambda = 0$. This is as valid a choice as any other in a pure gravity theory. Unfortunately, it is an unnatural one when one introduces any kind of matter. Indeed, set λ to zero, but assume that there is a nonvanishing vacuum (i.e., ground-state) energy: $\langle T_{\mu\nu}\rangle = -\langle\rho\rangle g_{\mu\nu}$; then the Einstein equation (1) reads

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -8\pi G_N T_{\mu\nu} + 8\pi G_N \langle\rho\rangle g_{\mu\nu} \tag{6}$$

The last term is interpreted as an effective cosmological constant:

$$\lambda_{\rm eff} = 8\pi G_N \langle \rho \rangle \equiv \frac{\Lambda^4}{m_P^2} \tag{7}$$

Generically, $\langle \rho \rangle$ receives a nonzero contribution from symmetry breaking: for instance, the scale Λ would be typically of the order of 100 GeV in the case of the gauge symmetry breaking of the Standard Model or 1 TeV in the case of supersymmetry breaking. But the constraint (5) now reads

$$\Lambda \leq 10^{-30}\, m_P \sim 10^{-3} \text{ eV} \tag{8}$$

It is this very unnatural fine tuning of parameters (in explicit cases $\langle \rho \rangle$ and thus Λ are functions of the parameters of the theory) that is referred to as the *cosmological constant problem*, or more accurately the vacuum energy problem.

2. THE ROLE OF SUPERSYMMETRY

If the vacuum energy is to be small, it may be easier to have it vanishing through some symmetry argument. Global supersymmetry is the obvious candidate. Indeed, the supersymmetry algebra

$$\{Q_r, \bar{Q}_s\} = 2\gamma^\mu_{rs} P_\mu \tag{9}$$

yields the following relation between the Hamiltonian $H = P_0$ and the supersymmetry generators Q_r:

$$H = \frac{1}{4} \sum_r Q_r^2 \tag{10}$$

and thus the vacuum energy $\langle 0|H|0\rangle$ is vanishing if the vacuum is supersymmetric ($Q_r|0\rangle = 0$).

Unfortunately, supersymmetry has to be broken at some scale since its prediction of equal mass for bosons and fermions is not observed in the physical spectrum. Then Λ is of the order of the supersymmetry breaking scale, that is, a few hundred GeV to 1 TeV.

However, the right framework to discuss these issues is supergravity, i.e., local supersymmetry, since locality implies here, through the algebra (9), invariance under "local" translations that are the diffeomorphisms of general relativity. In this theory, the graviton, described by the linear perturbations of the metric tensor $g_{\mu\nu}(x)$, is associated through supersymmetry with a spin-3/2 field, the gravitino ψ_μ. One may write a supersymmetric invariant combination of terms in the action

$$\mathcal{S} = \int d^4x\, \sqrt{g}[3m_P^2 m_{3/2}^2 - m_{3/2}\bar{\psi}_\mu \sigma^{\mu\nu}\psi_\nu] \tag{11}$$

where $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/4$. If the first term is made to cancel the vacuum energy, then the second term is interpreted as a mass term for the gravitino. We thus see that the criterion for spontaneous symmetry breaking changes from global supersymmetry (*nonvanishing vacuum energy*) to local supersymmetry or supergravity (*nonvanishing gravitino mass*). It is somewhat welcome news that a vanishing vacuum energy is not tightened to a supersymmetric spectrum. On the other hand, we have lost the only rationale that we had to explain a zero cosmological constant.

Let us recall for future use that, in supergravity, the potential for a set of scalar fields ϕ^i is written in terms of the Kähler potential $K(\phi^i, \overline{\phi}^{\bar{i}})$ [the normalization of the scalar field kinetic terms is simply given by the Kähler metric $K_{i\bar{j}} = \partial^2 K/\partial\phi^i\partial\overline{\phi}^{\bar{j}}$] and of the superpotential $W(\phi^i)$, a holomorphic function of the fields:

$$V = e^{K/m_P^2}\left[\left(W_i + \frac{K_i}{m_P^2}W\right)K^{i\bar{j}}\left(\overline{W}_{\bar{j}} + \frac{\overline{K}_{\bar{j}}}{m_P^2}\overline{W}\right) - 3\frac{|W|^2}{m_P^2}\right] + \text{D-terms} \tag{12}$$

where $K_i = \partial K/\partial\phi^i$, etc., and $K^{i\bar{j}}$ is the inverse metric of $K_{i\bar{j}}$. Obviously, the positive definiteness of the global supersymmetry scalar potential is lost in supergravity.

3. OBSERVATIONAL RESULTS

In recent years, there has been an increasing number of indications that the universe is presently undergoing accelerated expansion. This appears to be a strong departure from the standard picture of a matter-dominated universe. Indeed, the standard equation for the conservation of energy,

$$\dot{\rho} = -3(p + \rho)H \tag{13}$$

allows us to derive from the Friedmann equation (3), written in the case of a universe dominated by a component with energy density ρ and pressure p,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p) \tag{14}$$

Obviously, a matter-dominated ($\rho \sim 0$) universe is decelerating. One needs instead a component with a negative pressure.

A cosmological constant is associated with a contribution to the energy-momentum tensor as in (6) and (7):

$$T^\mu_\nu = -\Lambda^4\delta^\mu_\nu = (-\rho, p, p, p) \tag{15}$$

The associated equation of motion is therefore

$$p = -\rho \tag{16}$$

It follows from (14) that a cosmological constant tends to accelerate expansion.

The discussion of data is thus often expressed in terms of the energy density Λ^4 stored in the vacuum versus the energy density ρ_M in matter fields (baryons, neutrinos, hidden matter, . . .). It is customary to normalize with the critical density (corresponding to a flat universe):

$$\Omega_\Lambda = \frac{\Lambda^4}{\rho_c}, \qquad \Omega_M = \frac{\rho_M}{\rho_c}, \qquad \rho_c = \frac{3H^2}{8\pi G_N} \tag{17}$$

The relation

$$\Omega_M + \Omega_\Lambda = 1 \tag{18}$$

a prediction of many inflation scenarios, is found to be compatible with recent cosmic microwave background measurements [3].[2] It is striking that independent methods based on the measurement of different observables on rich clusters of galaxies all point toward a low value of $\Omega_M \sim 1/3$ [5]: mass-to-light ratio, baryon mass to total cluster mass ratio (the total baryon density in the universe being fixed by primordial nucleosynthesis), and cluster abundance. This necessarily implies a nonvanishing Ω_Λ (nonvanishing cosmological constant or a similar dynamical component).

There are indeed some indications going in this direction from several types of observational data. One which has been much discussed lately uses supernovae of type Ia as standard candles.[3] Two groups, the Supernova Cosmology Project [6] and the High-z Supernova Search [7], have found that distant supernovae appear to be fainter. If this is to have a cosmological origin, this means that, at fixed redshift, they are at larger distances and thus that the universe is accelerating.

More precisely, the relation between the flux f received on earth and the absolute luminosity $\mathscr{L}$ of the supernova depends on its redshift z, but also on the geometry of spacetime. Traditionally, flux and absolute luminosity are expressed on a log scale as apparent magnitude m_B and absolute magnitude M (magnitude is $-2.5 \log_{10}$ luminosity + const). The relation then reads

$$m_B = 5 \log(H_0 d_L) + M - 5 \log H_0 + 25 \tag{19}$$

The last terms are z independent *if one assumes that supernovae of type Ia are standard candles*; they are then measured by using low-z supernovae. The first term, which involves the luminosity distance d_L, varies logarithmi-

[2] This follows from the fact that the first acoustic peak is expected at an "angular" scale $l \sim 200/\sqrt{\Omega_M + \Omega_\Lambda}$ [44].

[3] By calibrating them according to the time scale of their brightening and fading.

cally with z up to corrections which depends on the geometry. Expanding in z,[4] one obtains [9]

$$H_0 d_L = cz\left[1 + \frac{1 - q_0}{2} z + \cdots \right] \tag{20}$$

where $q_0 \equiv -a\ddot{a}/\dot{a}^2$ is the deceleration parameter. This parameter is easily obtained from (14): in a spatially flat universe with only matter and a cosmological constant [cf. (16)], $\rho = \rho_M + \Lambda^4$ and $p = -\Lambda^4$, which gives

$$q_0 = \Omega_M/2 - \Omega_\Lambda \tag{21}$$

This allows us to put some limit on Ω_Λ on the model considered here. Let us note that the combination (21) is 'orthogonal' to the combination $\Omega_T \equiv \Omega_M + \Omega_\Lambda$ measured in CMB experiments (see footnote 3). The two measurements are therefore complementary: this is sometimes referred to as 'cosmic complementarity'.

Of course, such a type of measurement is sensitive to many possible systematic effects (evolution besides the light-curve time-scale correction, etc.), and this has fueled a healthy debate on the significance of present data. This obviously requires more statistics and improved quality of spectral measurements. A particular tricky systematic effect is the possible presence of dust that would dim supernovae at large redshift.

Other results come from gravitational lensing. The deviation of light rays by an accumulation of matter along the line of sight depends on the distance to the source [9]

$$r = \int_t^{t_0} \frac{dt}{a(t)} = \frac{1}{a(t_0)H_0}\left(z - \frac{1}{2}(1 + q_0)z^2 + \cdots\right) \tag{22}$$

and thus on the cosmological parameters Ω_M and Ω_Λ. As q_0 decreases (i.e., as the universe accelerates), there is more volume and more lenses between the observer and the object at redshift z. Several methods are used: abundance of multiply imaged quasar sources [10], strong lensing by massive clusters of galaxies (providing multiple images or arcs) [11], and weak lensing [12].

4. QUINTESSENCE

From the point of view of high-energy physics, it is difficult to imagine a rationale for a pure cosmological constant, especially if it is nonzero but small compared to the typical fundamental scales (electroweak, strong, grand

[4] Of course, since supernovae of redshift $z \sim 1$ are now being observed, an exact expression [8] must be used to analyze data. The more transparent form of (20) gives the general trend.

unified, or Planck scale). There should be physics associated with this form of energy and therefore dynamics. For example, in the context of string models, any dimensional parameter is expressed in terms of the fundamental string scale M_s and vacuum expectation values of scalar fields. The physics of the cosmological constant is then the physics of the corresponding scalar fields.

Introducing dynamics generally modifies the equation of state (16) to the more general form with negative pressure

$$p = w\rho, \qquad w < 0 \tag{23}$$

Let us recall that $w = 0$ corresponds to nonrelativistic matter (dust) and $w = 1/3$ corresponds to radiation. A network of light, nonintercommuting topological defects [13, 14], on the other hand, gives $w = -n/3$, where n is the dimension of the defect, i.e., 1 for a string and 2 for a domain wall. Finally, the equation of state for a minimaly coupled scalar field necessarily satisfies the condition $w \geq -1$.

Experimental data may constrain such a dynamical component just as it did with the cosmological constant. For example, in a spatially flat universe with only matter and an unknown component X with equation of state $p_X = w_X \rho_X$, one obtains (14) with $\rho = \rho_M + \rho_X$, $p = w_X \rho_X$ the following form for the deceleration parameter:

$$q_0 = \frac{\Omega_M}{2} + (1 + 3w_X)\frac{\Omega_X}{2} \tag{24}$$

where $\Omega_X = \rho_X/\rho_c$. Supernovae results give a constraint on the parameter w_X. Similarly, gravitational lensing effects are sensitive to this new component through (22).

A particularly interesting candidate in the context of fundamental theories is the case of a scalar field ϕ slowly evolving in a runaway potential which decreases monotonically to zero as ϕ goes to infinity [16, 17]. This is often referred to as *quintessence*. This can be extended to the case of a very light field (pseudo-Goldtone boson) which is presently relaxing to its vacuum state [15]. We will discuss the two situations in turn.

4.1. Runaway Quintessence

A runaway potential is frequently present in models where supersymmetry is dynamically broken. Indeed, supersymmetric theories are characterized by a scalar potential with many flat directions, i.e., directions ϕ in field space for which the potential vanishes. The corresponding degeneracy is lifted through dynamical supersymmetry breaking, that is, supersymmetry breaking through strong interaction effects. In some instances (dilaton or compactification radius), the field expectation value $\langle \phi \rangle$ actually provides the value of the strong interaction coupling. Then at infinite ϕ value, the coupling effectively goes to zero together with the supersymmetry-breaking effects and the flat direction is restored: the potential decreases monotonically to zero as ϕ goes to infinity.

Dynamical supersymmetry-breaking scenarios are often advocated because they easily yield the large-scale hierarchies necessary in grand unified or superstring theories in order to explain the smallness of the electroweak scale with respect to the fundamental scale. Let us take the example of supersymmetry breaking by gaugino condensation in effective superstring theories. The value g_0 of the gauge coupling at the string scale M_s is provided by the vacuum expectation value of the dilaton field s (taken to be dimensionless by dividing by m_P) present among the massless string modes: $g_0^2 = \langle s \rangle^{-1}$. If the gauge group has a one-loop beta function coefficient b, then the running gauge coupling becomes strong at the scale

$$\Lambda \sim M_s e^{-1/2bg_0^2} = M_s e^{-s/2b} \tag{25}$$

At this scale, the gaugino fields are expected to condense. Through dimensional analysis, the gaugino condensate $\langle \bar{\lambda}\lambda \rangle$ is expected to be of order Λ^3. Terms quadratic in the gaugino fields thus yield in the effective theory below condensation scale a potential for the dilaton

$$V \sim |\langle \bar{\lambda}\lambda \rangle|^2 \propto e^{-3s/b} \tag{26}$$

The s dependence of the potential is of course more complicated and one usually looks for stable minima with vanishing cosmological constant. But the behavior (26) is caracteristic of the large-s region and provides a potential slopping down to zero at infinity as required in the quintessence solution. A similar behavior is observed for moduli fields whose vev describes the radius of the compact manifolds which appear from the compactification from 10 or 11 dimensions to 4 in superstring theories [18].

Let us take therefore the example of an exponentially decreasing potential. More explicitly, we consider the action

$$\mathcal{S} = \int d^4x \sqrt{g} \left[-\frac{m_P^2}{2} R - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) \right] \tag{27}$$

which describes a real scalar field ϕ minimally coupled with gravity and the self-interactions of which are described by the potential

$$V(\phi) = V_0 e^{-\lambda \phi / m_P} \tag{28}$$

The energy density and pressure stored in the scalar field are, respectively,

$$\rho_\phi = \tfrac{1}{2}\dot{\phi}^2 + V(\phi), \qquad p_\phi = \tfrac{1}{2}\dot{\phi}^2 - V(\phi) \tag{29}$$

We will assume that the background (matter and radiation) energy density ρ_B and pressure p_B obey a standard equation of state

$$p_B = w_B \rho_B \tag{30}$$

If one neglects the spatial curvature ($k \sim 0$), the equation of motion for ϕ simply reads

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi} \tag{31}$$

with

$$H^2 = \frac{1}{3m_P^2}(\rho_B + \rho_\phi) \tag{32}$$

This can be rewritten as

$$\dot{\rho}_\phi = -3H\dot{\phi}^2 \tag{33}$$

We are looking for *scaling solutions*, i.e., solutions where the ϕ energy density scales as a power of the cosmic scale factor: $\rho_\phi \propto a^{-n_\phi}$ or $\dot{\rho}_\phi/\rho_\phi = -n_\phi H$. In this case, one easily obtains from (29) and (33) that the ϕ field obeys a standard equation of state

$$p_\phi = w_\phi \rho_\phi \tag{34}$$

with

$$w_\phi = \frac{n_\phi}{3} - 1 \tag{35}$$

Hence

$$\rho_\phi \propto a^{-3(1+w_\phi)} \tag{36}$$

If one can neglect the background energy ρ_B, then (32) yields a simple differential equation for $a(t)$ which is solved as

$$a \propto t^{2/[3(1+w_\phi)]} \tag{37}$$

Since $\dot{\phi}^2 = (1 + w_\phi)\rho_\phi$, one deduces that ϕ varies logarithmically with time. One then easily obtains from (31), (32) that

$$\phi = \phi_0 + (2/\lambda)\ln(t/t_0) \tag{38}$$

and

$$w_\phi = \lambda^2/3 - 1 \tag{39}$$

It is clear from (39) that, for λ sufficiently small, the field ϕ can play the role of quintessence.

But the successes of the standard big-bang scenario indicate that clearly ρ_ϕ cannot have always dominated: it must have emerged from the background energy density ρ_B. Let us thus now consider the case where ρ_B dominates. It turns out that the solution just discussed with $\rho_\phi \gg \rho_B$ and (39) is a late-time attractor [19] only if $\lambda^2 < 3(1 + w_B)$. If $\lambda^2 > 3(1 + w_B)$, the global attractor turns out to be a scaling solution [20, 21] with the following properties[5]:

$$\frac{\rho_\phi}{\rho_\phi + \rho_B} = \frac{3}{\lambda^2}(1 + w_B) \tag{40}$$

$$w_\phi = w_B \tag{41}$$

Equation (41) clearly indicates that this does not correspond to a quintessence solution (23).

Unfortunately, the semirealistic models discussed earlier tend to give large values of λ and thus the latter scaling solution as an attractor. For example, in the case (26) where the scalar field is the dilaton, $\lambda = 3/b$ with $b = C/(16/\pi^2)$ and $C = 90$ for E_8 gauge symmetry down to $C = 9$ for $SU(3)$. Moreover [21], on the observational side, the condition that ρ_ϕ should be subdominant during nucleosynthesis (in the radiation-dominated era) imposes that we take rather large values of λ. Typically, requiring $\rho_\phi/(\rho_\phi + \rho_B)$ to be smaller than 0.2 imposes $\lambda^2 > 20$.

Ways out of this problem have been proposed; I sketch some of them in turn.

One is the notion of *tracker field* [23]. This idea precisely rests on the existence of scaling solutions of the equations of motion which play the role of late-time attractors, as illustrated above. An alternative example is provided by a scalar field described by the action (27) with a potential

$$V(\phi) = \lambda \frac{\Lambda^{4+\alpha}}{\phi^\alpha} \tag{42}$$

with $\alpha > 0$. In the case where the background density dominates, one finds an attractor scaling solution [16, 24, 25, 22] $\phi \propto a^{3(1+w_B)/(2+\alpha)}$, $\rho_\phi \propto a^{-3\alpha(1+w_B)/(2+\alpha)}$. Thus ρ_ϕ decreases at a slower rate than the background density

[5] See ref. 22 for the case where the scalar field is nonminimally coupled to gravity.

($\rho_B \propto a^{-3(1+w_B)}$) and tracks it until it becomes of the same order at a given value a_Q. More precisely [18],

$$\phi = m_P \sqrt{\frac{\alpha(2+\alpha)}{3(1+w_B)}} \left(\frac{a}{a_Q}\right)^{3(1+w_B)/(2+\alpha)} \tag{43}$$

$$\rho_\phi \sim \lambda \frac{\Lambda^{4+\alpha}}{m_P^\alpha} \left(\frac{a}{a_Q}\right)^{-3\alpha(1+w_B)/(2+\alpha)} \tag{44}$$

One finds

$$w_\phi = -1 + \frac{\alpha(1+w_B)}{2+\alpha} \tag{45}$$

Shortly after ϕ has reached for $a = a_Q$ a value of order m_P, it satisfies the standard slow-roll conditions:

$$m_P |V'/V| \ll 1 \tag{46}$$

$$m_P^2 |V''/V| \ll 1 \tag{47}$$

and therefore (45) provides a good approximation to the present value of w_ϕ. Thus, at the end of the matter-dominated era, this field may provide the quintessence component that we are looking for.

Two features are particularly interesting in this respect. One is that this scaling solution is reached for rather general initial conditions, i.e., whether ρ_ϕ starts as of the same order or much smaller than the background energy density [23]. The second deals with the central question in this game: why is the ϕ energy density (or in the case of a cosmological constant, the vacuum energy density) emerging now? Since ϕ is of order m_P in this scenario, this can be rephrased as follows: why is $V(m_P)$ of the order of the critical energy density ϕ_c? Using (44), this amounts to a constraint on the parameters of the theory:

$$\Lambda \sim (H_0^2 m_P^{2+\alpha})^{1/(4+\alpha)} \tag{48}$$

For example, this gives for $\alpha = 2$, $\Lambda \sim 1$ GeV, not such an unnatural value.

On the other hand, we will see below that the fact that the present value for ϕ is of order m_P is a source of problems.

Models of dynamical supersymmetry breaking easily provide a model of the type just discussed [18]. Let us consider supersymmetric QCD with gauge group $SU(N_c)$ and $N_f < N_c$ flavors, i.e., N_f quarks Q^i (resp., antiquarks $\overline{Q}_i$), $i = 1, \ldots, N_f$, in the fundamental $\mathbf{N_c}$ (resp. antifundamental $\overline{\mathbf{N}}_\mathbf{c}$) of $SU(N_c)$. At the scale of dynamical symmetry breaking Λ where the gauge coupling becomes strong,[6] bound states of the meson type form: $\Pi^i{}_j = Q^i\overline{Q}_j$. The dynamics is described by a superpotential which can be computed nonperturbatively using standard methods:

$$W = (N_c - N_f)\,\frac{\Lambda^{(3N_c - N_f)/(N_c - N_f)}}{(\det \Pi)^{1/(N_c - N_f)}} \tag{49}$$

Such a superpotential has been used in the past, but with the addition of a mass or interaction term (i.e., a positive power of Π) in order to stabilize the condensate. One does not wish to do that here if Π is to be interpreted as a runaway quintessence component. For illustration purpose, let us consider a condensate diagonal in flavor space: $\Pi^i{}_j \equiv \phi^2\delta^i_j$ (see ref. 26 for a more complete analysis). Then the potential for ϕ has the form (42), with $\alpha = 2(N_c + N_f)/(N_c - N_f)$. Thus

$$w_\phi = -1 + \frac{N_c + N_f}{2N_c}(1 + w_B) \tag{50}$$

which clearly indicates that the meson condensate is a potential candidate for a quintessence component.

Another possibility for the emergence of the quintessence component out of the background energy density might be attributed to the presence of a local minimum (a "bump") in the potential $V(\phi)$: when the field ϕ approaches it, it slows down and ρ_ϕ decreases more slowly [n_ϕ being much smaller as w_ϕ temporarily becomes closer to -1; cf. (35)]. If the parameters of the potential are chosen carefully enough, this allows the background energy density, which scales as $a^{-3(1+w_B)}$, to become subdominant. This approach has recently been advocated by Albrecht and Skordis [27] in the context of an exponential potential. They argue quite sensibly that in a "realistic" string model, V_0 in (28) is ϕ dependent: $V_0(\phi)$. This new field dependence might be such as to generate a bump in the scalar potential and thus a local minimum. Since

$$\frac{1}{V}\frac{dV}{d\phi} = \frac{V_0'(\phi)}{V_0(\phi)} - \frac{\lambda}{m_P} \tag{51}$$

it suffices that $m_P V_0'(\phi)/V_0(\phi)$ become temporarily larger than λ in order to slow down the redshift of ρ_ϕ: once ρ_ϕ dominates, an attractor scaling solution of the type (38), (39) is within reach, if λ is not too large. As pointed out by Albrecht and Skordis, the success of this scheme does not require very small couplings.

[6] It is given by an expression such as (25) where g_0 is the value of the gauge coupling at the large scale M_s and b is the one-loop beta function coefficient for gauge group $SU(N_c)$.

One may note that in the preceding model one could arrange the local minimum in such a way as to completely stop the scalar field, allowing for a period of true inflation [28]. The last possibility that I will discuss goes one step further in this direction. It is known under several names: deflation [29], kination [30], quintessential inflation [31]. It is based on the remark that if a field ϕ is to provide a dynamical cosmological constant under the form of quintessence, it is a good candidate to account for an inflationary era where the evolution is dominated by the vacuum energy. In other words, are the quintessence component and the inflaton the same unique field?

In this kind of scenario, inflation (where the energy density of the universe is dominated by the ϕ-field potential energy) is followed by reheating where matter is created and by an era where the evolution is driven by the ϕ-field kinetic energy (which decreases as a^{-6}). Since matter energy density is decreasing more slowly ($\propto a^{-3}$), this turns into a matter-dominated era until the ϕ energy density eventually emerges as in the quintessence scenarios described above.

4.2. Pseudo-Goldstone Boson

There exists a class of models [15] very close in spirit to the case of runaway quintessence: they correspond to a situation where a scalar field has not yet reached its stable ground state and is still evolving in its potential.

More specifically, let us consider a potential of the form

$$V(\phi) = M^4 v\left(\frac{\phi}{f}\right) \tag{52}$$

where M is the overall scale, f is the vacuum expectation value $\langle \phi \rangle$, and the function v is expected to have coefficients of order one. If we want the potential energy of the field (assumed to be close to its vev f) to give a substantial fraction of the energy density at present time, we must set

$$M^4 \sim \rho_c \sim H_0^2 m_P^2 \tag{53}$$

However, requiring that the evolution of the field ϕ around its minimum has been overdamped by the expansion of the universe until recently imposes

$$m_\phi^2 = \frac{1}{2} V''(f) \sim \frac{M^4}{f^2} \leq H_0^2 \tag{54}$$

This is one of the slow-roll conditions familiar in the inflation scenarios.

From (53) and (54), we conclude that f is of order m_P (as the value of the field ϕ in runaway quintessence) and that $M \sim 10^{-3}$ eV [not surprisingly, this is the scale Λ typical of the cosmological constant; see (8)]. The field

ϕ must be very light: $m_\phi \sim h_0 \times 10^{-60}\, m_P \sim h_0 \times 10^{-33}$ eV. Such a small value is only natural in the context of an approximate symmetry: the field ϕ is then a pseudo-Goldstone boson.

A typical example of such a field is provided by the axion field (QC Daxion [32] or string axion [33]). In this case, the potential simply reads

$$V(\phi) = M^4[1 + \cos(\phi/f)] \tag{55}$$

5. QUINTESSENTIAL PROBLEMS

However appealing, the quintessence idea is difficult to implement in the context of realistic models [34, 35]. The main problem lies in the fact that the quintessence field must be extremely weakly coupled to ordinary matter. This problem can take several forms:

1. We have assumed until now that the quintessence potential monotonically decreases to zero at infinity. In realistic cases, this is difficult to achieve because the couplings of the field to ordinary matter generate higher order corrections that are increasing with larger field values, unless forbidden by a symmetry argument. For example, in the case of the potential (42), the generation of a correction term $\lambda_d \phi^d$ puts in jeopardy the slow-roll constraints on the quintessence field unless very stringent constraints are imposed on the coupling λ_d [35].

Similarly, because the viev of ϕ is of order m_P, one must take into account the full supergravity corrections. One may then argue [36] that this could put in jeopardy the positive-definiteness of the scalar potential, a key property of the quintessence potential. This may point toward models where $\langle W \rangle = 0$ [but not its derivatives; see (12)] or to no-scale type models: in the latter case, the presence of three moduli fields T^i with Kähler potential $K = -\Sigma_i \ln(T^i + \overline{T}^i)$ cancels the negative contribution $-3|W|^2$ in (12).[7]

2. the quintessence field must be very light [34]. If we return to our example of supersymmetric QCD in (42), $V''(m_P)$ provides an order of magnitude for the mass of the quintessence component:

$$m_\phi \sim \Lambda \left(\frac{\lambda}{m_P} \right)^{1+\alpha/2} \tag{56}$$

If we take as above $\alpha = 2$ and $\Lambda \sim 1$ GeV, we obtain a mass of order 10^{-36}

[7] Moreover, supergravity corrections may modify some of the results. For example, the presence of a (flat) Kähler potential in (12) induces exponential field-dependent factors. A more adequate form for the inverse power law potential (42) is thus [36] $V(\phi) = \lambda[\exp(\phi^2/2M_P^2)]\Lambda^{4+\alpha}/\phi^\alpha$. The exponential factor is not expected to change much the late-time evolution of the quintessence energy density. Brax and Martin [36] argue that it changes the equation of state through the value of w_ϕ.

GeV $\sim \hbar c/(10^{20}$ m). Similarly, we have seen that the mass of a pseudo-Goldstone boson that could play the role of quintessence is typically $H_0 \sim 10^{-33}$ eV. This field must therefore be very weakly coupled to matter; otherwise its exchange would generate observable long-range forces. Eötvös-type experiments put very severe constraints on such couplings.

Again, for the case of supersymmetric QCD, higher order corrections to the Kähler potential of the type

$$\kappa(\phi_i, \phi_j^\dagger)\left[\beta_{ij}\left(\frac{Q^\dagger Q}{m_P^2}\right) + \text{h.c.}\right] \tag{57}$$

will generate couplings of order one to the standard matter fields ϕ_i, $\phi_j^\dagger$ since $\langle Q\rangle$ is of order m_P. In order to alleviate this problem, Masiero *et al.* [26] proposed a solution much in the spirit of the least coupling principle of Damour and Polyakov [37]: the different functions β_{ij} have a *common* minimum close to the value $\langle Q\rangle$. In the early stages of the evolution of the universe, when $Q \ll m_P$, couplings of the type (57) generate a contribution to the mass of the Q field which, being proportional to H, does not spoil the tracker solution.

3. It is difficult to find a symmetry that would prevent any coupling of the form $\beta(\phi/m_P)^n F^{\mu\nu}F_{\mu\nu}$ to the gauge field kinetic term. Since the quintessence behavior is associated with time-dependent values of the field of order m_P, this would generate, in the absence of fine tuning, corrections of order one to the gauge coupling. But the time dependence of the fine structure constant, for example, is very strongly constrained [38]: $|\dot{\alpha}/\alpha| < 5 \times 10^{-17}$ year^{-1}. This yields a limit [34]

$$|\beta| \leq 10^{-6}\, m_P H_0/\langle\dot{\phi}\rangle \tag{58}$$

where $\langle\dot{\phi}\rangle$ is the average over the last 2×10^9 years.

A possible solution is to implement an approximate continuous symmetry of the type $\phi \to \phi + \text{const}$ [34]. This symmetry must be approximate since it must allow for a potential $V(\phi)$. Such a symmetry would only allow derivative couplings, an example of which is an axion-type coupling $\tilde{\beta}(\phi/m_P)F^{\mu\nu}\tilde{F}_{\mu\nu}$. If $F_{\mu\nu}$ is the color $SU(3)$ field strength, QCD instantons yield a mass of order $\tilde{\beta}\Lambda_{\text{QCD}}/m_P$, much too large to satisfy the preceding constraint. In any case, supersymmetry would relate such a coupling to the coupling $\beta(\phi/m_P)F^{\mu\nu}F_{\mu\nu}$ that we started out to forbid.

The very light mass of the quintessence component points toward scalar–tensor theories of gravity, where such a (Brans–Dicke) scalar field is found. This has triggered some recent interest in this type of theory. Attractor solutions have been found for non-minimally coupled fields [22, 39]. However, as discussed above, one problem is that scalar–tensor theories lead to time-varying constants of nature. One may either put some limit on the couplings of the scalar field [40] or use the attractor mechanism toward general relativity that was found by Damour and Nordtvedt [41, 37]. For instance, in the solution proposed by Bartolo and Pietroni [42], the quintessence component is first attracted to general relativity and then to a standard tracker solution.

All the preceding shows that there is extreme fine tuning in the couplings of the quintessence field to matter unless they are forbidden by some symmetry. This is somewhat reminiscent of the fine tuning associated with the cosmological constant. In fact, the quintessence solution does not claim to solve the cosmological constant (vacuum energy) problem described above. If we take the example of a supersymmetric theory, the dynamical cosmological constant provided by the quintessence component clearly does not provide enough supersymmetry breaking to account for the mass difference between scalars (sfermions) and fermions (quarks and leptons): at least 100 GeV. There must be other sources of supersymmetry breaking and one must fine tune the parameters of the theory in order not to generate a vacuum energy that would completely drown ρ_ϕ.

The new and exciting properties of theories with extra dimensions may provide a framework for attacking this more ambitious problem of why the corrections to the vacuum energy are not observed in the cosmological constant [43, 44].

In any case, the quintessence solution shows that, once this fundamental problem is solved, one can find explicit fundamental models that effectively provide the small amount of cosmological constant that seems required by experimental data.

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